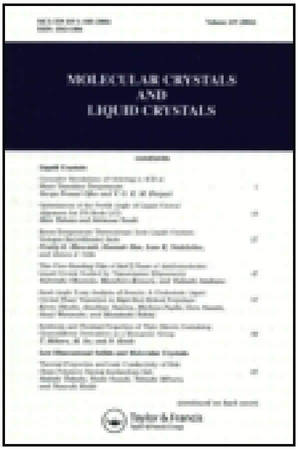

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Light-Induced Threshold Instability of Nematic Director in Cylindrical Waveguide

Mikhailo Lednei [a] & Igor Pinkevich [a]
[a] Physics Faculty, Kyiv Taras Shevchenko University, Prospect Glushkova 6, Kyiv, 03022, Ukraine

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Light-Induced Threshold Instability of Nematic Director in Cylindrical Waveguide

MIKHAILO LEDNEI and IGOR PINKEVICH

Physics Faculty, Kyiv Taras Shevchenko University, Prospect Glushkova 6, Kyiv 03022, Ukraine

The dependence of director reorientation threshold on the parameters of electromagnetic wave in waveguide as well as director boundary conditions on the waveguide surface is obtained.

Keywords: liquid crystal; waveguide; threshold instability

INTRODUCTION

In recent time the interest to possibility of effective control by transmitting of electromagnetic signals in waveguides loaded with nematic liquid crystals (NLC) has appeared[1,2]. In present paper we consider a director instability threshold in such system in the field of electromagnetic wave propagating inside a waveguide.

GENERAL EXPRESSIONS FOR ELECTROMAGNETIC FIELD

Let us consider an infinite round cylindrical waveguide of radius R with ideal metallic surface filled with nonmagnetic NLC. We assume the boundary conditions for NCL director at the waveguide surface to be homogeneous ones so the director spatial distribution only depends on the distance r from the waveguide axis. Solving Maxwell's equations in waveguide with anisotropic dielectric core one can get the expressions for travelling TE- and TM-waves. Putting for simplicity the waveguide waves to be symmetrical ones we can get the next expressions for nonzero components of TE-waves

$$H_z^l = A_l J_0\left(\mu_l^{(1)} \frac{r}{R}\right), \quad H_r^l = -A_l \frac{ik_l R}{\mu_l^{(1)}} J_1\left(\mu_l^{(1)} \frac{r}{R}\right),$$

$$E_\varphi^l = A_l \frac{i\mu_0 \omega R}{\mu_l^{(1)}} J_1\left(\mu_l^{(1)} \frac{r}{R}\right), \quad \frac{\omega^2}{c^2}\varepsilon_{\varphi\varphi} - k_l^2 = \left(\frac{\mu_l^{(1)}}{R}\right)^2 \qquad (1)$$

and TM-waves

$$E_z^l = A_l J_0\left(\mu_l^{(0)} \frac{r}{R}\right), \quad E_r^l = -A_l \frac{ik_l R\varepsilon_{zz}}{\mu_l^{(0)}\varepsilon_{rr}} J_1\left(\mu_l^{(0)} \frac{r}{R}\right),$$

$$H_\varphi^l = -A_l \frac{i\omega R\varepsilon_0\varepsilon_{zz}}{\mu_l^{(0)}} J_1\left(\mu_l^{(0)} \frac{r}{R}\right), \quad \frac{\varepsilon_{zz}}{\varepsilon_{rr}}\left(\frac{\omega^2}{c^2}\varepsilon_{rr} - k_l^2\right) = \left(\frac{\mu_l^{(0)}}{R}\right)^2. \qquad (2)$$

Here l denotes the mode of electromagnetic wave, $J_\nu(x)$ is the Bessel function, $\mu_l^{(\nu)}$ is a root of number l of Bessel function, A_l is an amplitude, ε_{ik} is a dielectric constant, ω is a frequency and k_l is a wave vector of l-th mode. In (1), (2) a multiplier $\exp\left[i\left(k_l z - \omega t\right)\right]$ is omitted.

THRESHOLD VALUE OF ELECTROMAGNETIC FLUX

Free energy of NLC in waveguide takes a form:

$$F = F_{el} + F_E + F_S\,, \qquad F_{el} = \frac{K}{2}\iint\left[(div\,\vec{n})^2 + (rot\,\vec{n})^2\right]dV\,,$$

$$F_E = -\frac{1}{16\pi}\sum_l \int \varepsilon_{ik} E_i^l E_k^{l*}\, dV\,, \qquad F_S = -\frac{W}{2}\int(\vec{n}\vec{e}_s)^2\, dS\,,\ W > 0\,. \quad (3)$$

Here F_{el} is Frank's elastic energy in one constant approximation (K_{13} and K_{24} terms are omitted for simplicity), $\vec{n}$ denotes a NCL director, F_E is a contribution of electromagnetic field into NCL free energy[3], the summation in F_E is over all modes of electromagnetic field; F_S is a director anchoring energy with waveguide surface chosen in the form of Rapini potential[4], $\vec{e}_s$ is a unit vector along the easy orientation axis at the waveguide surface.

Assume the unit easy vector $\vec{e}_s$ to be parallel the waveguide axis and therefore a director $\vec{n}||OZ$ in the absence of electromagnetic field.

i) Let the only symmetric TE-wave propagates along the waveguide. TE-wave possesses only electric component $E_\varphi \neq 0$ and one can wait the director threshold reorientation in the $(\vec{e}_\varphi, \vec{e}_z)$-plane ($\vec{e}_z$ is unit vector of cylindrical frame). Thus we must seek for a director in the form $\vec{n} = \sin\theta(r,t)\cdot\vec{e}_\varphi + \cos\theta(r,t)\cdot\vec{e}_z$, where $\theta(r,t)$ is an angle of director with its initial direction along $\vec{e}_z$. Minimizing free energy (3) with respect to the angle θ one can get in linear in θ approximation the next differential equation and corresponding boundary condition

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} - \left(\frac{1}{r^2} - \frac{\varepsilon_a}{8\pi K}\sum_l \left|E_\varphi^l\right|^2\right)\theta = \eta\frac{\partial\theta}{\partial t} \qquad (4)$$

$$\left[r\frac{\partial\theta}{\partial r}+(1+\xi)\theta\right]_{r=R}=0\,, \tag{5}$$

where $\eta \sim (10^{-2} \div 1)P$, $\xi = WR/K$, $\varepsilon_{ik} = \varepsilon_{\perp}\delta_{ik} + \varepsilon_a n_i n_k$, $\varepsilon_a = \varepsilon_{\parallel} - \varepsilon_{\perp} > 0$, $\varepsilon_{\parallel}$, $\varepsilon_{\perp}$ is a dielectric anisotropy. The solution of equation (4) one can seek in the form

$$\theta(r,t) = f(r)\exp\left(\alpha t/\eta\right). \tag{6}$$

The director grows with time if $\alpha > 0$. Then threshold value of electric field can be found from the condition $\alpha = 0$. The solution $f(r)$ in the equation (6) we can seek in the form

$$f(r) = \sum_{m=1}^{\infty} C_m J_1\left(\mu_m \frac{r}{R}\right), \tag{7}$$

where μ_m is a root of equation

$$\mu J_1'(\mu) + (1+\xi)J_1(\mu) = 0\,, \tag{8}$$

and the coefficients C_m satisfy the next system of homogeneous equations

$$\sum_{m=1}^{\infty} B_{km} C_m = 0\,, \quad k = 1,2,3,\ldots. \tag{9}$$

Here

$$B_{km} = \int_0^1 F(x)\,J_1(\mu_k x)\,J_1(\mu_m x)\,x\,dx\,,$$

$$F(x) = \frac{\varepsilon_a}{8\pi K}(\mu_0 c R A_1 \Omega)^2 \cdot \sum_l \left[S_l J_1\left(\mu_l^{(1)} x\right)/\mu_l^{(1)}\right]^2 - \delta_{km}\cdot(\mu_k)^2$$

and is cumbersome enough to be presented here, $S_l = A_l / A_1$, $\Omega = \omega R/c$. The condition for nontrivial solution of the system of homogeneous equations (9) yields the equation to determine the threshold value of amplitude A_1 at the given values of S_l (e.g. ratio of the amplitude of l -th mode to the amplitude of the first mode).

From equations (9) it follows that series (7) converges with respect to m absolutely and uniformly. Therefore we can restrict ourselves to a finite number of terms in (7) within any necessary accuracy. This would result in a finite number of equations (9) and thus the equation for A_1 becomes finite and can be solved numerically.

To simplify the numerical calculations we took only into account in the series (7) the terms with $m \leq 3$ and have determined the threshold value of A_1 and then the threshold value of electromagnetic energy flux through waveguide cross-section

$$P_{th} = \frac{1}{2}\varepsilon_0 c^2 \sum_l \iint \left[\vec{E}^l, \vec{B}^{l*}\right]_z dS =$$

$$= \pi\mu_0 c R^3 A_1^2 \Omega \cdot \sum_l k_l \left(S_l / \mu_l^{(1)}\right)^2 \cdot \int_0^1 J_1^2\left(\mu_l^{(1)} x\right) x \, dx \, .$$

On Figure 1 we plotted the dimensionless threshold electromagnetic flux $\Pi_{TE} = \frac{\varepsilon_a}{8\pi^2 K}\mu_0 c P_{th}$ versus the director anchoring energy parameter ξ at the single mode regime of TE-wave. The similar dependence $\Pi_{TE}(\xi)$ takes place for two-mode regime of TE-wave too.

On Figure 2 the value Π_{TE} versus electromagnetic frequency Ω is shown for the cases of one and two mode regimes. It is seen that, in general, Π_{TE} increases with increase of frequency Ω but it decreases at the appearing of the second mode. Such behaviour of Π_{TE} is conditioned by frequency dependence of electric and magnetic components of TE-wave (see (1)). Actually, the director reorientation is only determined by E_φ component, while Π_{TE} is determined by both E_φ- and H_r-components. Ratio H_r/E_φ increases with increase of frequency and it leads to the corresponding increase of Π_{TE}. As the second mode appears its magnetic energy has minimal value ($H_r^l \sim k_l$) and due to this the value of Π_{TE} decreases. Evidently, the more is part of electromagnetic energy connected with second TE-mode the more is decrease of Π_{TE}.

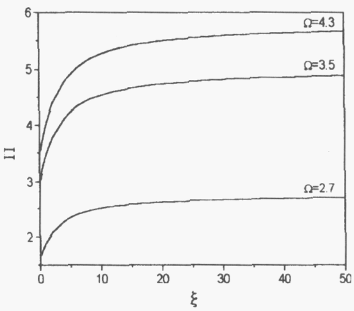

FIGURE 1.

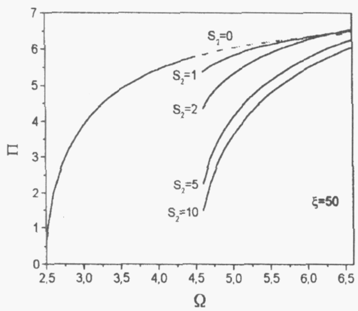

FIGURE 2.

ii) Let now the only symmetric TM-wave propagates along the waveguide. This wave has E_z and E_r electric components and director can reorientate in $(\vec{e}_z, \vec{e}_r)$-plane. After the same procedure as for TE-wave one can get for the function $\theta(r,t)$ the equation (4) in which $\left|E_\varphi^l\right|^2$ is replaced by $\left(\left|E_r^l\right|^2 - \left|E_z^l\right|^2\right)$. Analogous calculations lead to the $\Pi_{TM}(\xi)$ which is qualitatively the same as shown on Figure 1 for TE-wave. But the dependence $\Pi_{TM}(\xi)$ is completely different (see Figure

3). In the case of TM-wave the electric part of energy in Π_{TM} increases more quickly with increase of frequency than its magnetic part and due to this the value of Π_{TM} decreases with increase of Ω. As the second mode appears the reorienting electric field in this mode has minimal value ($E_r^l \sim k_l$, $k_2 \approx 0$) and Π_{TM} must increase as shown on Figure 3. The more is part of electromagnetic energy in the second TM-mode the more is increase of Π_{TM}. To reorientate a director the $\left|E_r^l\right|^2$ must dominate over $\left|E_z^l\right|^2$ and it leads to the condition $\Omega \to \Omega_{th}$. At $\Omega \to \Omega_{th}$ the $\Pi_{TM} \to \infty$ as one can see from Figure 3.

Assume now the easy vector $\vec{e}_s$ to be parallel to the vector $\vec{e}_r$ and thus in the initial state the director $\vec{n} || \vec{e}_r$.

i) If TE-wave propagates along waveguide the director can reorientate in the plane $(\vec{e}_r, \vec{e}_\varphi)$. In this case $\Pi_{TE}(\xi)$ and $\Pi_{TE}(\Omega)$ have the same qualitative behaviour as shown on Figures 1, 2.

ii) In the case of TM-mode a director can reorientate from direction $\vec{e}_r$ to $\vec{e}_z$ that is contrary to the considered above case of director reorientation from $\vec{e}_z$ to $\vec{e}_r$. Therefore the frequency dependence of Π_{TM} is now opposite to that shown on Figure 3, namely, $\Pi_{TM}(\Omega)$ increases with increase of Ω (see Figure 4). The dependence $\Pi_{TM}(\xi)$ leaves qualitatively the same as in previous cases.

Assume now $\vec{e}_s || \vec{e}_\varphi$ and thus the initial state of a director $\vec{n} || \vec{e}_\varphi$. TE-wave has only E_φ component and therefore cannot reorientate a director. TM-wave has E_z and E_r electric components and can reorientate a director in $(\vec{e}_\varphi, \vec{e}_z)$- or in $(\vec{e}_\varphi, \vec{e}_r)$-planes. In the first case $\Pi_{TM}(\Omega)$ is similar to that shown on Figure 4 and in the second case to that shown on Figure 3 but for both cases there are no threshold values for Ω where $\Pi_{TM}(\Omega) \to \infty$. $\Pi_{TM}(\xi)$ has the same view as on Figure 1.

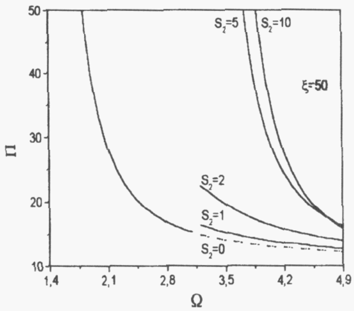

FIGURE 3.

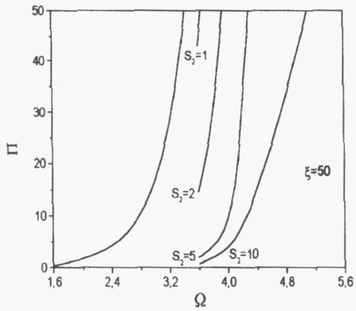

FIGURE 4.

CONCLUSIONS

Threshold value of electromagnetic energy flux in waveguide P_{th} depends on the initial director orientation, director anchoring energy with waveguide surface, frequency and type of electromagnetic wave, distribution of electromagnetic energy between wave modes.

If director anchoring energy increases the P_{th} increases as well. In the case of TE-wave the P_{th}, in general, increases with increase of electromagnetic wave frequency. But at the appearance of the second TE-mode the more of electromagnetic energy is connected with the second mode the more P_{th} decreases. In the case of TM-wave the threshold value P_{th} increases or decreases with increase of electromagnetic wave frequency in dependence of the initial director orientation and the plane of director reorientation.

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